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Generalized Modal Shock Spectra Method for Spacecraft Loads Analysis

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Unlike the traditional shock spectra approach, the generalization presented in this paper permits elastic interaction between the spacecraft and launch vehicle in order to obtain accurate bounds on the spacecraft response and structural loads. In addition, the modal response from a previous launch vehicle transient analysis—with or without a dummy spacecraft—is exploited in order to define a modal impulse as a simple idealization of the actual forcing function. The idealized modal forcing function is then used to derive explicit expressions for an estimate of the bound on the spacecraft structural response and forces. Greater accuracy is achieved with the present method over the traditional shock spectra, while saving much computational effort over the transient analysis.

Introduction

In designing spacecraft structures, it is desirable to perform several iterative cycles of inexpensive loads analyses to account for the frequent changes occurring in the spacecraft. Performing a complete transient model that includes the spacecraft-launch vehicle composite for each cycle of loads is costly, time consuming, and difficult to implement with respect to schedule. In an effort to circumvent this problem, this paper describes a shock spectra method as an inexpensive alternative to the usual complete transient analysis. The method is a generalization of an earlier approach presented in Ref. 1, and results in significant improvements.

In the traditional shock spectra concept, a shock spectrum $S(\omega,\xi)$ of a function $\ddot{X}_0(t)$ is the largest peak response (a bound) of a single-degree-of-freedom oscillator subjected to the base input $\ddot{X}_0(t)$. The implicit assumption is made that the input $\ddot{X}_0(t)$ is not affected by the presence of the oscillator. This is true only if the mass of the oscillator is infinitesimal. In removing this assumption and extending the shock spectra concepts to spacecraft loads analysis, the present approach makes use of the following observations and assumptions:

1) The general objective is to avoid an overall analysis of the launch vehicle and the new spacecraft composite structure.

2) The goal of the structural designer is to determine maxima or bounds of the response quantities of interest rather than time histories. Since maxima are also the objective of the shock spectra concept, a shock spectra approach is readily applicable to structural design.

3) The spacecraft-launch vehicle interface is statically determinate.

4) No external forces are acting directly upon the spacecraft other than through the common spacecraft-launch vehicle interface.

5) The mass and inertia ratio of the spacecraft to that of the launch vehicle is small, but not infinitesimal. This assumption can easily be removed.

6) In any loads analysis—whether based on a transient, shock spectra, or any other approach—two basic ingredients are present; a model idealizing the dynamic environment, and another idealizing the composite structural system. The nature of each model is influenced by the selected approach and is constrained by cost and time.

7) There has been a previous modal analysis done for the launch vehicle with a dummy spacecraft mass representation or another simple spacecraft, the results of which are used for the iterative design of a new spacecraft. In the derivation of the present approach, it is assumed that the available launch vehicle modal analysis was done without any spacecraft at the spacecraft-launch vehicle interface, i.e., with an unloaded interface. In general, however, this will not be the case. If the launch vehicle modal analysis has been made with a rigid spacecraft of mass $[M_0]$, this mass can be easily subtracted from the residual mass $[M_{ii}]_{RES}$ defined later with no other change in the method. If the launch vehicle modal analysis has been made with an elastic spacecraft, the effect of that elastic spacecraft on the launch vehicle modes can be removed according to the procedure described in Appendix C of Ref. 2, thereby producing launch vehicle modes without any spacecraft. These are then used in the analysis described herein.

8) The actual launch vehicle forcing functions, usually unknown to the spacecraft analyst, may be idealized by a simpler form in order to obtain an explicit closed-form solution. A complete definition of this idealization is possible from the previous analysis of item 7, and is done at the modal level.

In view of the preceding remarks, the present approach consists of the following:

First, the modal forcing function $F_\ell(t)$, representing the launch vehicle modal contribution of an actual flight event, is modeled by an equivalent launch vehicle modal forcing function $F_\ell^e(t)$. The equivalent forcing function emphasizes the view that the shape of the response time history is of no consequence, and that only a bound on the response is of interest to the designer. The equivalency between the actual forcing function and the idealized one is established—not on the basis of producing identical response time histories—but on the basis of producing identical peak of the shock spectra of the launch vehicle modal response derived from the previous launch vehicle analysis discussed under item 7.

Second, in considering the composite structural system which consists of a spacecraft modeled by S-normal modes and a launch vehicle modeled by L-normal modes, there will be $(S+L)$ modally coupled equations of motion. Unlike the

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transient analysis where the solution is expressed in the complete $(S+L)$ space of modal coordinates and time, a bound on the complete solution is established herein by

1) Idealizing the totality of $(S+L)$ mathematical space of modal coordinates by an array of nested $(S\times L)$ mathematical subspaces, in each of which only one spacecraft mode is coupled with one launch vehicle mode. To derive a bound on the total solution in the original $(S+L)$ mathematical space, an explicit solution in the form of spacecraft modal response time history $q_{s\ell}(t)$ is first derived for the pair of modes in a typical subspace. An expression of the bound $Q_{s\ell}$ is then established.

2) Numerical computations are made first to establish the bounds $Q_{s\ell}$ on each of the $(S\times L)$ discrete modal responses. To account for unknown design tolerances and variations in the structural model idealization, worst cases are provided by allowing realistic possible tuning between the spacecraft and the launch vehicle modes.

3) Next, a bound on the total spacecraft modal response Q_s is constructed by summation over all the discrete L-bounds $Q_{s\ell}$ for that spacecraft mode.

4) Finally, spacecraft displacements, accelerations, and member loads are obtained by adding the contributions of all spacecraft modes.

Basic Equations

The equations of motion of a damped linear structural system under the action of prescribed forces $\{F_j(t)\}$ and accelerations $\{\ddot{R}(t)\}$ can be found in the literature.[3] For each normal mode $n=1,2,...N$, the equations are

$$M_{nn}(\ddot{q}_n+2\xi_n\omega_n\dot{q}_n+\omega_n^2 q_n)=\langle\phi_{nj}\rangle\{F_j\}-\langle m_{nR}\rangle\{\ddot{R}\} \quad (1)$$

where

q_n = generalized modal coordinates that are related to the physical degrees-of-freedom $\{u_b\}$ by $\{u_b\}=[\phi_{bn}]\{q_n\}$. For rigid body modes, the subscript R is used so that q_R designates rigid body generalized coordinates.

$[\phi_{bn}]$ = n-normal mode shapes associated with q_n for a structure subjected to R statically determinate set of constraints.

ξ_n = percent of critical modal damping associated with q_n.

ω_n = natural frequency associated with q_n.

$M_{nn}=\langle\phi_{nb}\rangle[m_{bb}]\{\phi_{bn}\}$ = elements of the diagonal mass matrix, where normalization is subsequently chosen so that $M_{nn}=1$. Here, $[m_{bb}]$ represent the physical masses at the physical degrees-of-freedom b in the model.

$\langle m_{nR}\rangle=\langle\phi_{nb}\rangle\ [m_{bb}]\ [\phi_{bR}]$ = mass coupling between an elastic mode n and the rigid body mode R.

$[\phi_{bR}]$ = R-rigid body mode shapes associated with q_R due to a unit deformation imposed upon the Rth constraint, one at a time.

$\{F_j\}$ = vector of forces applied to the launch vehicle at location j.

Equation (1) can be applied to a spacecraft mounted on a launch vehicle through a statically determinate interface, as shown in Fig. 1. Using subscripts s, ℓ, and i to designate quantities associated, respectively, with the spacecraft, launch vehicle, and their common interface i, Eq. (1) becomes:

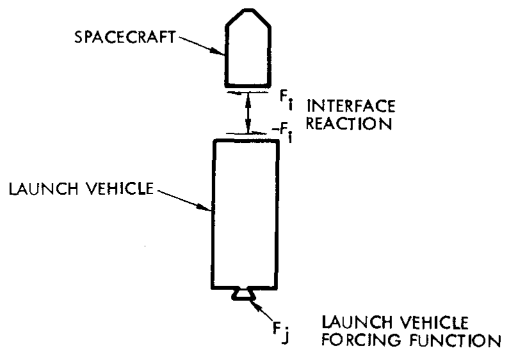

Fig. 1 Composite system of spacecraft and launch vehicle.

1) for the ℓth normal mode of the free-free launch vehicle to which is added the new spacecraft through the statically determinate interface at i

$$\ddot{q}_\ell+2\xi_\ell\omega_\ell\dot{q}_\ell+\omega_\ell^2 q_\ell=\langle\phi_{\ell j}\rangle\{F_j\}-\langle\phi_{\ell i}\rangle\{F_i\} \quad (2)$$

and 2) for the sth normal mode of the spacecraft, cantilevered from i,

$$\ddot{q}_s+2\xi_s\omega_s\dot{q}_s+\omega_s^2 q_s=-\langle m_{si}\rangle\{\ddot{R}_i\} \quad (3)$$

In Eqs. (2) and (3), the interface acceleration $\{\ddot{R}_i\}$ is obtained by superposition of contributions of all launch vehicle modes, rigid and elastic

$$\{\ddot{R}_i\}=[\phi_{iR}]\{\ddot{q}_R\}+[\phi_{i\ell}]\{\ddot{q}_\ell\} \quad (4)$$

The launch vehicle rigid body accelerations $\{\ddot{q}_R\}$ above are obtained as a special case of Eq. (2) when $\omega_\ell=0$. Designating the total launch vehicle mass matrix by $[M_{RR}]$, Eq. (2) gives

$$[M_{RR}]\{\ddot{q}_R\}=[\phi_{Rj}]\{F_j\}-[\phi_{Ri}]\{F_i\} \quad (5)$$

Furthermore, the reaction forces $\{F_i\}$ on the statically determinate spacecraft to launch vehicle interface can be obtained by

$$\{F_i\}=[M_{ii}]\{\ddot{R}_i\}+[m_{is}]\{\ddot{q}_s\},\quad s=1,2,...,\bar{S} \quad (6)$$

When considering all spacecraft modes $\bar{S}$,

$$[M_{ii}]=[\phi_{ib}][m_{bb}][\phi_{bi}] \quad (7)$$

is the total mass matrix of the spacecraft with respect to the interface degrees-of-freedom i, and $[\phi_{bi}]$ represent all spacecraft rigid body modes with respect to i. The matrix $[M_{ii}]$ is a geometric definition of mass, center of mass, and inertias, independent of any modal consideration.

By solving Eqs. (4-6) for $\{\ddot{R}_i\}$ and $\{F_i\}$, one obtains

$$\{\ddot{R}_i\}=([I]+[\psi_{ii}][M_{ii}])^{-1}([\psi_{ij}]\{F_j\}+[\phi_{i\ell}]\{\ddot{q}_\ell\}-[\psi_{ii}][m_{is}]\{\ddot{q}_s\}) \quad (8a)$$

$$\{F_i\}=([I]+[M_{ii}][\psi_{ii}])^{-1}([M_{ii}][\psi_{ij}]\{F_j\}+[M_{ii}][\phi_{i\ell}]\{\ddot{q}_\ell\}+[m_{is}]\{\ddot{q}_s\}) \quad (8b)$$

where

$$[\psi_{ii}]=[\phi_{iR}][M_{RR}{}^{-1}][\phi_{Ri}]$$

$$[\psi_{ij}]=[\phi_{iR}][M_{RR}{}^{-1}][\phi_{Rj}]$$

For a composite system consisting of a spacecraft modeled by $\bar{S}$ degrees-of-freedom and a launch vehicle modeled by $\bar{L}$ degrees-of-freedom, there will be $(\bar{S}+\bar{L})$ system of coupled simultaneous differential equations. Equations (2) and (3) represent a typical pair of these $(\bar{S}+\bar{L})$ equations. The coupling arises as a consequence of $\{\ddot{R}_i\}$ and $\{F_i\}$ of Eq. (8), since the spacecraft modes and the launch vehicle modes have been obtained using two separate models rather than one integrated model. All previous equations contain no approximation, and up to that stage of analysis all modes are retained.

Generalized Modal Displacement Spectra

A. Idealized Equations

A number of assumptions and simplifications will now be introduced.

Truncation

The total number of degrees-of-freedom $\bar{S}$ and $\bar{L}$ are usually very large, and are often dictated by requirements to obtain accurate stress distribution in the spacecraft and launch vehicle structures. However, in all modal approaches, including the conventional modal transient solution, the $(\bar{S}+\bar{L})$ degrees-of-freedom are replaced by a set of modal coordinates. It is not practical or desirable to retain all $(\bar{S}+\bar{L})$ modal coordinates associated with the $(\bar{S}+\bar{L})$ degrees-of-freedom. The original $(\bar{S}+\bar{L})$ degrees-of-freedom are truncated to a smaller set of modes $(S+L)<(\bar{S}+\bar{L})$, so that the cost of performing a dynamic analysis can be substantially reduced with little loss in the accuracy of the computed stresses, accelerations, and displacements.

In the following, the launch vehicle model is known through its normal modes, for which a truncation is assumed to have been performed by the launch vehicle analyst and no further truncation for the launch vehicle will be performed. Therefore, it will be assumed here that $L=\bar{L}$. For the spacecraft, however, only a selected set of modes $S<\bar{S}$ will be retained. The truncated modes $(\bar{S}-S)$ can be accounted for as far as the interface forces are concerned by placing their equivalent residual mass $[M_{ii}]_{\rm RES}$ at the interface. The residual mass is defined as follows.

First, for the spacecraft cantilevered from i, each normal mode s has an effective mass matrix $[m^s_{ii}]$ with respect to the interface i given by[4]

$$[m^s_{ii}]=\{m_{is}\}\langle m_{si}\rangle \tag{9}$$

This effective mass matrix $[m^s_{ii}]$ gives the physical mass and its location with respect to the interface that is needed to physically model each mode s.

If all $\bar{S}$ modes are retained, then the sum of all the effective masses of Eq. (9) will be equal to the total mass matrix $[M_{ii}]$ of Eq. (7) defined from geometry and mass distribution, i.e.,

$$[M_{ii}]=[\phi_{ib}][m_{bb}][\phi_{bi}]=\sum_{s=1}^{\bar{S}}[m^s_{ii}] \tag{10}$$

If not all modes are retained, the summation on $[m^s_{ii}]$ will not yield the total mass $[M_{ii}]$, and a residual mass $[M_{ii}]_{\rm RES}$ can be defined by

$$[M_{ii}]_{\rm RES}=[M_{ii}]-\sum_{s=1}^{S}[m^s_{ii}],\quad S\le\bar{S} \tag{11}$$

This residual mass $[M_{ii}]_{\rm RES}$ is then placed at the interface i so that the launch vehicle is loaded by a spacecraft model that accounts for the total spacecraft mass.

Attaching the residual mass to the spacecraft-launch vehicle interface will change the normal modes $\phi_{i\ell}$ of the launch vehicle. A new eigenvalue solution for the launch vehicle carrying the residual mass may be performed to obtain a new set of launch vehicle modal coordinates if desired. However, this expense can be avoided with negligible loss of accuracy. Because the spacecraft residual mass defined above is typically much smaller than the launch vehicle total mass, an approximation may be invoked wherein the off-diagonal terms are neglected in the coupled modal equations of the launch vehicle loaded with the residual mass at the interface. The resulting uncoupled equation for a typical launch vehicle mode—ℓ becomes

$$(1+M_{s\ell})\ddot{q}_\ell+2\xi_\ell\omega_\ell\dot{q}_\ell+\omega_\ell^2q_\ell=\langle\phi_{\ell j}\rangle\{F_j\}-\langle\phi_{\ell i}\rangle\{F_i\} \tag{12}$$

where

$$M_{s\ell}\simeq\langle\phi_{\ell i}\rangle[M_{ii}]_{\rm RES}\{\phi_{i\ell}\}$$

Equation (12) is now a modification of Eq. (2) to account for the residual mass of the spacecraft.

Modal Pairing

The usefulness of Eqs. (11) and (12) depends on the approach employed in the subsequent steps. If a conventional transient analysis is used, the form of Eqs. (2), (3), and (8) does not change and the truncated $(S+L)$ coupled modes are simultaneously considered. This solution yields time histories of the response quantities of interest. However, since bounds rather than time histories are needed for design, much computational effort can be saved by introducing mathematical idealizations that allow a relatively simple estimate of the bounds. This is achieved by idealizing the entire $(S+L)$ mathematical space of modal coordinates by an array of nested $(S\times L)$ discrete mathematical subspaces. In each subspace, a spacecraft mode s is paired with a launch vehicle mode ℓ, thus giving rise to $(S\times L)$ possible pairings. A bound on the true solution in the original $(S+L)$ mathematical space is established by summation over the individual discrete bounds in the $(S\times L)$ subspaces. A typical subspace consisting of a spacecraft mode s and a launch vehicle mode ℓ may be physically viewed as in Fig. 2. The governing equations are Eqs. (3) and (12), except that $\{\ddot{R}_i\}$ and $\{F_i\}$ of Eq. (8) are now modified to include only one spacecraft and one launch vehicle mode of the retained $(S+L)$ modes. The interface acceleration and reaction for such (s,ℓ) subspace are $\{\ddot{R}^{s\ell}_i\}$ and $\{F^{s\ell}_i\}$, respectively. Also, the response of the spacecraft mode is $q_{s\ell}$. Thus,

$$\{\ddot{R}^{s\ell}_i\}=([I]+[\psi_{ii}][m^s_{ii}])^{-1}([\psi_{ij}]\{F_j\}+\{\phi_{i\ell}\}\ddot{q}_\ell-[\psi_{ii}]\{m_{is}\}\ddot{q}_{s\ell}) \tag{13a}$$

$$\{F^{s\ell}_i\}=([I]+[m^s_{ii}][\psi_{ii}])^{-1}([m^s_{ii}][\psi_{ij}]\{F_j\}+[m^s_{ii}]\{\phi_{i\ell}\}\ddot{q}_\ell+\{m_{is}\}\ddot{q}_{s\ell}) \tag{13b}$$

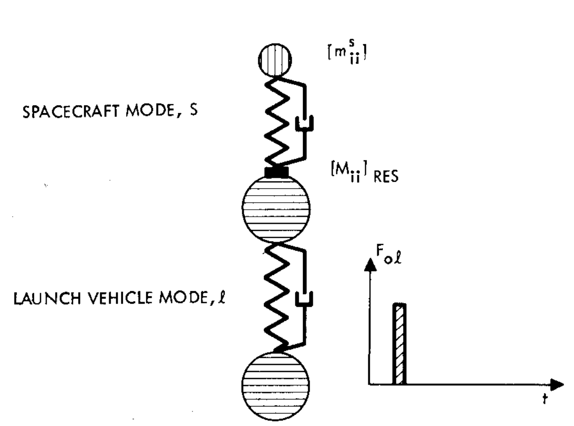

Fig. 2 Representation of subspace (S,ℓ) for a pair of spacecraft mode and launch vehicle mode.

Considering Eq. (13), if the ratio of the Euclidian norm of the mass matrix $[m^s_{ii}]$ defined by Eq. (9) for one spacecraft mode to the norm of the total launch vehicle mass matrix $[M_{RR}]$ is small, simplifying approximations can be introduced in which

$$[m^s_{ii}][\psi_{ii}] = [m^s_{ii}][\phi_{iR}][M_{RR}{}^{-1}][\phi_{Ri}] \doteq 0 \tag{14a}$$

$$[m^s_{ii}][\psi_{ij}] = [m^s_{ii}][\phi_{iR}][M_{RR}{}^{-1}][\phi_{Rj}] \doteq 0 \tag{14b}$$

$$[\psi_{ii}][m_{is}] = [\phi_{iR}][M_{RR}{}^{-1}][\phi_{Ri}][m_{is}] \doteq 0 \tag{14c}$$

This results in

$$\{\ddot{R}^{s\ell}_i\} \cong [\psi_{ij}]\{F_j\} + \{\phi_{i\ell}\}\ddot{q}_\ell \tag{15a}$$

$$\{F^{s\ell}_i\} \cong [m^s_{ii}]\{\phi_{i\ell}\}\ddot{q}_\ell + \{m_{is}\}\ddot{q}_s \tag{15b}$$

which when substituted into Eqs. (3) and (12) yields the following simplified version of the equations of motion for an arbitrary pair of coupled spacecraft and launch vehicle modes

$$\left[\begin{array}{c|c} (1+\mu_{s\ell}) & \mu_{s\ell} \\ \hline \mu_{s\ell} & \mu_{s\ell} \end{array}\right] \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \left[\begin{array}{c|c} 2\xi_1\omega_1 & 0 \\ \hline 0 & 2\xi_2\omega_2\mu_{s\ell} \end{array}\right] \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix}$$

$$+ \left[\begin{array}{c|c} \omega_1^2 & 0 \\ \hline 0 & \omega_2^2\mu_{s\ell} \end{array}\right] \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} F_\ell(t) \\ 0 \end{Bmatrix} \tag{16}$$

The following definitions have been used in Eq. (16):

$$m_{s\ell} = \langle\phi_{\ell i}\rangle\{m_{is}\}\langle m_{si}\rangle\{\phi_{i\ell}\}, \quad \mu_{s\ell} = \frac{m_{s\ell}}{1+M_{s\ell}}$$

$$\omega_1 = \frac{\omega_\ell}{\sqrt{1+M_{s\ell}}}, \quad \xi_1 = \frac{\xi_\ell}{\sqrt{1+M_{s\ell}}}, \quad \omega_2 = \omega_s, \quad \xi_2 = \xi_s$$

$$F_\ell(t) = \langle\phi_{\ell j}\rangle\{F_j(t)\}, \quad q_\ell = \left(\frac{1}{1+M_{s\ell}}\right)x_1$$

$$q_{s\ell} = \left(\frac{\langle m_{si}\rangle\{\phi_{i\ell}\}}{1+M_{s\ell}}\right)x_2 = \left(\frac{\pm\sqrt{m_{s\ell}}}{1+M_{s\ell}}\right)x_2 \tag{17}$$

Note that using Eqs. (13) instead of Eqs. (15) will remove assumption 5 and will simply result in redefining the terms of Eq. (16).

B. Modal Displacement Bound

Equation (16) describes the motion in any of the idealized $(S\times L)$ subspaces (see Fig. 2) where only one spacecraft mode is coupled with one launch vehicle mode. A bound on the spacecraft generalized modal displacement in such subspace is derived from an explicit time-dependent solution of Eq. (16) in which the actual launch vehicle modal forcing function $F_\ell(t)$ is modeled by an equivalent forcing function having a simpler form of variation with time. The selected idealization of the forcing function is an impulse delta function having a magnitude $F_{0\ell}$ or, alternatively, an initial velocity with a magnitude $v_{0\ell}$.

An analog computer solution of Eq. (16) and the details of the analytical solution to the same equations are given in Ref. 2. The analytical expressions for $x_1(t)$ and $x_2(t)$ are

$$x_1(t) = \frac{v_{0\ell}}{(\Omega_2^2-\Omega_1^2)}\left[\frac{\Omega_2^2(1-\Omega_1^2)}{\bar{\omega}_{n1}}e^{-\xi_{n1}\omega_{n1}t}\sin\bar{\omega}_{n1}t\right.$$
$$\left.-\frac{\Omega_1^2(1-\Omega_2^2)}{\bar{\omega}_{n2}}e^{-\xi_{n2}\omega_{n2}t}\sin\bar{\omega}_{n2}t\right] \tag{18a}$$

$$x_2(t) = \frac{R^2 v_{0\ell}}{(\Omega_2^2-\Omega_1^2)}\left[\frac{1}{\bar{\omega}_{n1}}e^{-\xi_{n1}\omega_{n1}t}\sin\bar{\omega}_{n1}t\right.$$
$$\left.-\frac{1}{\bar{\omega}_{n2}}e^{-\xi_{n2}\omega_{n2}t}\sin\bar{\omega}_{n2}t\right] \tag{18b}$$

The following parameters have been used in Eqs. (18):

$$\omega_{n1} = \omega_2\Omega_1, \quad \omega_{n2} = \omega_2\Omega_2, \quad \Omega_1 = \Omega_0 - \frac{\Delta\Omega}{2}, \quad \Omega_2 = \Omega_0 + \frac{\Delta\Omega}{2}$$

$$\Omega_0 = \tfrac{1}{2}\sqrt{(R+1)^2+\mu_{s\ell}}, \quad \Delta\Omega = \sqrt{(R-1)^2+\mu_{s\ell}}$$

$$R = \omega_1/\omega_2 = \Omega_1\Omega_2, \quad \bar{\omega}_{n1} = \omega_{n1}\sqrt{1-\xi^2_{n1}}, \quad \bar{\omega}_{n2} = \omega_{n2}\sqrt{1-\xi^2_{n2}}$$

$$\xi_{n1}\omega_{n1} + \xi_{n2}\omega_{n2} = 2\omega_2\beta, \quad \xi_{n1}\omega_{n1} - \xi_{n2}\omega_{n2} = 2\omega_2\theta$$

$$\beta = \tfrac{1}{2}[\xi_1 R + (1+\mu_{s\ell})\xi_2]$$

$$\theta = \left[\frac{\xi_1 R(1-\Omega_1^2)^2 + \mu_{s\ell}\xi_2\Omega_1^4}{(1-\Omega_1^2)^2+\mu_{s\ell}} - \frac{\xi_1 R + (1+\mu_{s\ell})\xi_2}{2}\right] \tag{19}$$

The maximum value of $x_2(t)$ is defined as the "generalized" displacement shock spectra since the "base acceleration" $\ddot{x}_1(t)$ is allowed to be affected by $\ddot{x}_2(t)$, unlike the traditional shock spectra concept. In order to derive relatively simple expressions for the generalized modal displacement spectra, further minor mathematical simplifications will be introduced in Eqs. (18) and (19). These simplifications were tested and found accurate when the results below were compared with the results of the analog solution.

First, since ξ^2_{n1} and ξ^2_{n2} are much smaller than unity, the approximation

$$\xi_{n1} \simeq \xi_{n2} \simeq \xi_n = (\xi_{n1}+\xi_{n2})/2, \quad \text{for } R\neq 0 \tag{20a}$$

$$\xi_n = \tfrac{1}{2}\xi_{n2}, \quad \text{for } R=0 \tag{20b}$$

is used for the damped natural frequencies $\bar{\omega}_{n1}$ and $\bar{\omega}_{n2}$, so that

$$\bar{\omega}_{n1} \simeq \omega_2(\rho_1-\rho_2), \quad \bar{\omega}_{n2} \simeq \omega_2(\rho_1+\rho_2) \tag{21}$$

where

$$\rho_1 = \Omega_0\sqrt{1-\xi^2_n}, \quad \rho_2 = (\Delta\Omega/2)\sqrt{1-\xi^2_n}$$

The approximation of Eqs. (20) is not used for the exponential part of the solution.

Furthermore, according to Eqs. (19), $\theta \ll \beta$, especially in the neighborhood of the frequency ratio $R=1$. Therefore, it is sufficient to retain only the first-order approximation for $e^{\theta t}$. This approximation, along with that of Eqs. (20), leads to the following expressions:

$$x_2(\tau) \cong -\omega_1 v_{0\ell}[D_1(\tau)+D_2(\tau)], \quad \tau = \omega_2 t \tag{22}$$

where

$$D_1(\tau) = \frac{1}{2\omega_2^2\rho_2}\left(\frac{\rho_2}{\rho_1}-\theta\tau\right)e^{-\beta\tau}\sin\rho_1\tau\cos\rho_2\tau \tag{23a}$$

$$D_2(\tau) = \frac{-1}{2\omega_2^2\rho_1}\left(\frac{\rho_1}{\rho_2}-\theta\tau\right)e^{-\beta\tau}\cos\rho_1\tau\sin\rho_2\tau \tag{23b}$$

For $R\neq 0$, since ρ_2 is smaller than ρ_1, an accurate estimate of the maximum value of $x_2(\tau)$ of Eq. (22) is provided by the discrete maxima $D_{\max}$ of the simple sinusoid, where $\cos\rho_1\tau$ and $\cos\rho_2\tau$ are either zero or unity. Therefore, $D_{\max}$ is given by the largest of

$$D_{1\max} = \frac{1}{2\omega_2^2 \rho_2}\left(\frac{\rho_2}{\rho_1} - \theta\bar{\tau}\right)e^{-\beta\bar{\tau}}\sin\rho_1\bar{\tau} \tag{24a}$$

$$D_{2\max} = \frac{1}{2\omega_2^2 \rho_1}\left(\frac{\rho_1}{\rho_2} - \theta\bar{\tau}\right)e^{-\beta\bar{\tau}}\sin\rho_2\bar{\tau} \tag{24b}$$

where $\bar{\tau}$ is the smallest root of the characteristic equations:

For $D_{1\max}$:

$$\tan\rho_1\bar{\tau} = \frac{\rho_1}{\beta}\,\frac{[1-(\rho_1/\rho_2)\theta\bar{\tau}]}{[1+(\rho_1/\rho_2)(\theta/\beta)(1-\beta\bar{\tau})]} \tag{25a}$$

For $D_{2\max}$:

$$\tan\rho_2\bar{\tau} = \frac{\rho_2}{\beta}\,\frac{[1-(\rho_2/\rho_1)\theta\bar{\tau}]}{[1+(\rho_2/\rho_1)(\theta/\beta)(1-\beta\bar{\tau})]} \tag{25b}$$

The definitions introduced in Eqs. (17) and the equalities of Eqs. (24) and (25) are combined to give a bound on the generalized modal displacement $Q_{s\ell}$

$$Q_{s\ell} = \left|\frac{\omega_1 v_{0\ell}\sqrt{m_{s\ell}}}{1+M_{s\ell}}D_{\max}\right| \tag{26}$$

Equations (24-26) represent a generalization of the traditional shock spectra concept. They also represent an improvement over the results of Ref. 1 in that they hold true for an arbitrary pair of coupled launch vehicle mode and spacecraft mode, while taking into account the proximity of their respective natural frequencies. This makes it possible in the present approach to include systematically contributions of all launch vehicle and spacecraft modes in the computation of bounds on the mode displacements and, subsequently, on the member loads. It is noted here that Eqs. (24) suggest that the damping effect is through the "average" damping β of the spacecraft and the launch vehicle as defined in Eqs. (19) rather than through each damping separately. Although derived for $R \neq 0$, Eq. (26) is still valid in the limit for $R \to 0$ (i.e., when the spacecraft elastic modes are paired with the launch vehicle rigid body modes). This is shown in Ref. 2.

The generalized shock spectra will be identical to the traditional shock spectra in the limit when, in Eq. (26), the mass ratio $\mu_{s\ell}$ defined by Eqs. (17) for the oscillator equivalent to a spacecraft mode, is allowed to approach zero while the associated stiffness increases such that the natural frequency is held unaltered. This is the case of the shock spectrum of a launch vehicle mode. This remark will be useful, subsequently, in evaluating the initial velocity $v_{0\ell}$ of Eq. (26).

C. Determination of the Initial Modal Velocity $v_{0\ell}$

An analysis of the launch vehicle and a representation of the spacecraft is usually done by the launch vehicle organization for the structural design of the launch vehicle. In order to properly design the part of the launch vehicle at the interface with the spacecraft, the reaction loads from the spacecraft to the launch vehicle must be introduced in such an analysis. Since these loads are almost entirely due to the fundamental cantilevered modes of the spacecraft, a simple model of the spacecraft having as few as six degrees-of-freedom is adequate. As a minimum, a simple spacecraft model with three degrees-of-freedom provides the required information at the spacecraft-launch vehicle interface. This simple spacecraft model is used for the design of the launch vehicle at its interface with the spacecraft, but is not intended to be used for the detail design of the spacecraft. However, this simple model can be used to check the launch vehicle data and to determine the weighting factor defined later.

The unknown initial modal velocity $v_{0\ell}$ can be determined from the requirement that the "actual" and the "idealized" forcing functions produce identical bounds for the modal displacement spectra. Information for the "actual" forcing function can be obtained either from the analysis done on a simplified model of the spacecraft-launch vehicle combination, or from recorded flight response data for another spacecraft. In case of an available simple model analysis, the knowledge of the modal response time histories for each mode leads to the best estimate of $v_{0\ell}$. For this case, the peak value of the traditional displacement shock spectra D_ℓ^P is calculated for the modal acceleration response $\ddot{q}_\ell(t)$ of the launch vehicle. It should be noted that D_ℓ^P will need be computed only once for the launch vehicle mode and each flight event of interest, as long as the launch vehicle and the forcing functions are not changed.

Estimate of $v_{0\ell}$ from the Modal Response

Assuming now that the time histories $\ddot{q}_\ell(t)$, $\ell = 1,2,...L$ for the launch vehicle are available from a previous launch vehicle analysis, and that $D_\ell^P(\omega_2,\xi_2)$ representing the peak of the modal displacement spectrum $D_\ell(\omega_2,\xi_2)$ for each mode has been evaluated, then $v_{0\ell}$ can be determined by requiring that D_ℓ^P be equal to its counterpart Q_ℓ^P derived from Eq. (26) for $\mu_{s\ell} \to 0$. This is illustrated in Fig. 3. To derive Q_ℓ^P for $R \neq 0$, the shock spectrum Q_ℓ is computed for each launch vehicle elastic mode $\ell = 7,8,...L$ from Eq. (26) in the limit as $\mu_{s\ell} \to 0$. Reference 2 gives the proper values of the modal shock spectra for the case when the frequency ratio $R = 0$, i.e., for launch vehicle rigid body modes $\ell = 1,...6$.

$$Q_\ell = |\omega_1 v_{0\ell} \lim_{\mu_{s\ell}\to 0}(D_{\max})| \tag{27}$$

The peak value Q_ℓ^P occurs for $R = R_\ell^P$. Finding the exact value of R^P that maximizes Q_ℓ is rather involved analytically. Only an estimate is given here. Considering the special case when θ and ξ_n^2 are assumed negligibly small, one finds

$$\lim_{\mu_{s\ell}\to 0}(D_{\max}) = R\left\{\exp\left[-\frac{\tan^{-1}(a_r/2\beta)}{(a_r/2\beta)}\right]\Big/\omega_2^2\sqrt{a_r^2+4\beta^2}\right\}$$

$$\text{for } r = 1,2 \tag{28}$$

where

$$a_r = \begin{cases}(R+1) & \text{for } r=1\\ (R-1) & \text{for } r=2\end{cases}$$

By inspection of Eq. (28), the peak occurs for $r = 2$ and $R = 1$. The exact solution was evaluated from the analog simulation in Ref. 2 in which the maxima Q_ℓ^P occurred for $R = 1$. For

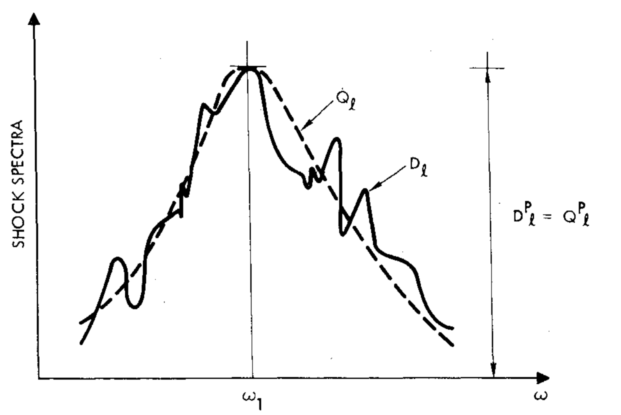

Fig. 3 Relationship between modal displacement spectra D_ℓ and Q_ℓ for launch vehicle modes.

$R=1$, Eq. (27) yields the following expression for the product $\omega_1 v_{0\ell}$:

$$\omega_1 v_{0\ell} = [e\omega_1^2(\xi_1+\xi)]D_\ell^P(\omega_2,\xi) \tag{29}$$

where $e=2.718$.

Instead of the modal displacement peak $D_\ell^P(\omega_2,\xi)$, the modal acceleration peak $A_\ell^P(\omega_2,\xi)$ may be used so that

$$\omega_1 v_{0\ell} = e(\xi_1+\xi)A_\ell^P(\omega_2,\xi) \qquad \text{for } R\neq 0 \tag{30a}$$

$$\omega_1 v_{0\ell} = A_\ell^P \qquad \text{for } R=0 \tag{30b}$$

where ξ is the damping used in calculating the shock spectra of $\ddot{q}_\ell(t)$.

Estimate of $v_{0\ell}$ from Physical Interface Response

In the absence of availability of $\ddot{q}_\ell(t)$, a set of initial modal velocities $v_{0\ell}$ can also be obtained in a manner similar to that of Ref. 5, where it is assumed that the time histories of a number of degrees-of-freedom $\bar{N}$ at the launch vehicle-spacecraft interface are known for another spacecraft or a dummy one from an analysis or flight data. Next, it is assumed that the mode shapes ϕ_n of the structure corresponding to the available interface acceleration are also known. Then, each mode is given an initial velocity v_{0n}, and the total interface acceleration is calculated from

$$\ddot{R}_i(t) = -\sum_{n=1}^{\bar{N}} \omega_n v_{0n}\phi_n e^{-\xi_n\omega_n t}(a_n \sin\omega_d t + 2\xi_n\cos\omega_d t) \tag{31}$$

where

$$a_n = (1-2\xi_n^2)/\sqrt{1-\xi_n^2} \quad \text{and} \quad \omega_d = \omega_n\sqrt{1-\xi_n^2}$$

The value of v_{0n} can be determined by trial and error so that the shock spectra of $\ddot{R}_i(t)$ envelopes the shock spectra of the real response. Since only a small number of vehicle modes are significantly contributing to $\ddot{R}_i(t)$, one can choose a set of modes $N<\bar{N}$ to simplify the calculations. The important criterion is that v_{0n} and the modes retained give an envelope of the real shock spectra.

Finally, the initial modal velocity $v_{0\ell}$ for the launch vehicle is obtained by the transformation of Eq. (32) where a bound for $v_{0\ell}$ may be obtained by taking root-sum-square values.

$$\omega_1 v_{0\ell} \simeq \sqrt{\sum_n (V_{\ell n}\omega_n v_{0n})^2}; \quad \ell=1,\ldots L \tag{32}$$

where $V_{\ell n}$ are the mode shapes associated with the eigenvalue solution to remove the elastic dummy spacecraft, as shown in Appendix C of Ref. 2.

D. Tuning

During the early stages of design, the spacecraft and launch vehicle modes and frequencies are obtained from analyses that usually contain a large degree of uncertainty. In order to account for such uncertainties, one may introduce an artificial tuning between the spacecraft and launch vehicle modes. Unlike other methods, the present approach makes artificial tuning possible because Eq. (26) is valid for any pair of spacecraft and launch vehicle modes, regardless of the degree of proximity of their respective frequencies.

Two forms of artificial tuning have been identified—global and local. In the global tuning, the entire spectrum of launch vehicle frequencies is incrementally scaled in either direction relative to the spacecraft frequency spectrum. For each increment, a global response

$$\bar{Q} = \sqrt{\sum_\ell\sum_s Q_{s\ell}^2} \tag{33}$$

is computed and used as a measure for determining the worst case for design purpose. In this scheme, tuning is achieved by finding the scaling quantity that maximizes $\bar{Q}$. Clearly, limits on the allowable relative scaling must be selected in advance and the search for the maximum $\bar{Q}$ conducted within these limits.

In the local tuning, the response is maximized for each spacecraft mode, one at a time. This is achieved by allowing the nearest launch vehicle frequency to coincide with that of the spacecraft frequency under consideration, (i.e., $R\rightarrow 1$) provided that the two were originally separated by no more than a preselected amount. Other schemes for tuning can be also devised.

Spacecraft Relative Displacement, Absolute Acceleration, and Member Loads

A. Relative Displacements

Bound estimates on the relative displacement associated with any degree-of-freedom b in the spacecraft model can be obtained by first establishing a bound Q_s in terms of the modal displacement spectra $Q_{s\ell}$ of Eq. (26). Since each $Q_{s\ell}$ results from coupling between a spacecraft mode s and only one launch vehicle mode ℓ, contributions from all launch vehicle modes L should be included. In absence of the time variable in Eq. (26), time phasing among the modes is not retained, but an estimate of the bound can be provided by summation of the absolute values or in the rss sense. Thus,

$$Q_s \cong \sum_{\ell=1}^{L} |Q_{s\ell}| \tag{34a}$$

or

$$Q_s \cong \sqrt{\sum_{\ell=1}^{L} Q_{s\ell}^2} \tag{34b}$$

or

$$Q_s \cong \sqrt{\sum_{\ell=1}^{L} [W_\ell(\omega_s,\omega_\ell)Q_{s\ell}]^2} \tag{34c}$$

where $W_\ell(\omega_s,\omega_\ell)$ is a weighting function introduced here to account for time phasing between the launch vehicle modes. This weighting function can be determined from the available transient response of the simplified model of the spacecraft or from matching of the shock spectra of the launch vehicle interface.

Using one of the expressions in Eqs. (34), a bound on the relative displacements $\{D_{bs}\}$ is found from

$$\{D_{bs}\} = \{\phi_{bs}\}Q_s \tag{35}$$

Similarly, by summation over the spacecraft modes for absolute values or in the rss sense, one obtains a bound on the total relative displacements $\{D_b\}$

$$\{D_b\} \cong \left\{\sum_{s=1}^{S} |D_{bs}|\right\} \tag{36a}$$

or

$$\{D_b\} \cong \sqrt{\sum_{s=1}^{S} \{D_{bs}^2\}} \tag{36b}$$

or

$$\{D_b\} \cong \sqrt{\sum_{s=1}^{S} \{(W_s(\omega_s)D_{bs})^2\}} \tag{36c}$$

B. Absolute Acceleration

Referring to Eq. (3), the maximum absolute acceleration $\{a_b\}$ for a degree-of-freedom b on the spacecraft results from the interface acceleration $\{\ddot{R}_i\}$ plus the relative acceleration

$\{\ddot{q}_s\}$ due to flexibilities of the spacecraft structure

$$\{a_b\} = [\phi_{bs}]\{\ddot{q}_s\} + [\phi_{bi}]\{\ddot{R}_i\} \tag{37}$$

In order to evaluate $\{a_b\}$, one may compute a bound $\bar{Q}_{s\ell}$ for the relative accelerations $\ddot{q}_{s\ell}$ in a manner similar to the bound $Q_{s\ell}$ of Eq. (26) for relative displacements. However, because of the complexity of the algebra involved, the relative displacement bound $Q_{s\ell}$ may be used instead of $\bar{Q}_{s\ell}$, provided that a proper correction is introduced to account for mode truncation errors.

From Eq. (3), the maximum of $\{\ddot{q}_s\}$ will occur approximately when $\{\dot{q}_s\} = 0$, so that when substituted into Eq. (37)

$$\{a_b\} \simeq -[\phi_{bs}][\omega_s^2]\{q_s\} - [\phi_{bs}][m_{si}]\{\ddot{R}_i\} + [\phi_{bi}]\{\ddot{R}_i\} \tag{38}$$

Using the orthogonality condition $[\phi_{sb}][m_{bb}][\phi_{bs}] = [I]$, it can be shown that when all spacecraft modes $\bar{S}$ are used in Eq. (38), the second and third terms of the right-hand side cancel each other, and

$$\{a_b\} = -[\phi_{bs}][\omega_s^2]\{q_s\}, \quad s = 1,2,\ldots\bar{S} \tag{39}$$

i.e., the total absolute acceleration, including the base input, can be expressed in terms of all the modal displacements alone. On the other hand, if a truncated set of modes $S < \bar{S}$ are used, Eq. (38) should be fully employed. Now, replacing $\{q_s\}$ by the bound $\{Q_s\}$, one obtains

$$\{a_b\} = \| [\phi_{bs}][\omega_s^2]\{Q_s\} \| + \underline{\| ([\phi_{bi}] - [\phi_{bs}][m_{si}])\{\ddot{R}_{is}\} \|} \tag{40}$$

where $s = 1,2,\ldots S \leq \bar{S}$, and the double bar $\|\ldots\|$ means that a bound is taken and that a summation over the spacecraft modes is invoked in a manner similar to Eqs. (36).

The combined use of Eqs. (14), (15), and (30) permits the approximation

$$\{\ddot{R}_{is}\} \simeq [\phi_{i\ell}]\{\omega_I v_{0\ell}\} \tag{41}$$

The underlined term in Eq. (40) is a correction that accounts for truncation of the spacecraft modes. It approaches zero as more modes are taken.

C. Member Loads

If the modal displacement method is used to calculate the member forces $\{F_a\}$, one might write

$$\{F_a\} = [C_{ab}][\phi_{bs}]\{Q_s\}, \quad s = 1,2,\ldots,\bar{S} \tag{42}$$

where $[C_{ab}]$ = matrix of force coefficients, whose elements are the ath force component associated with a displacement in the bth degree-of-freedom.

Here, again, because the modal displacement method can be sensitive to mode truncation errors, the same underlined correction of Eq. (40) can be introduced. Thus, by replacing $[\phi_{bs}]\{Q_s\}$ of Eq. (42) by its equivalent $[1/\omega_s^2]\{a_b\}$ obtained from Eq. (40), a more accurate expression for $\{F_a\}$ can be used

$$\{F_a\} = \| [C_{ab}][\phi_{bs}]\{Q_s\} \| + \underline{\| [C_{ab}]\left[\frac{1}{\omega_s^2}\right]([\phi_{bi}] - [\phi_{bs}][m_{si}])\{\ddot{R}_{is}\} \|} \tag{43}$$

where $s = 1,2,\ldots S \leq \bar{S}$, and the underlined term is the member load correction due to mode truncations. As before, the $\|\ldots\|$ means that a bound is taken similar to Eqs. (36) during summation over the spacecraft modes.

Conclusions

The primary drawback of the original shock spectra method (Ref. 1) is that it was very dependent upon analyst judgment. However, it had the advantage of low cost of analysis and yielded bounds that could be readily used for design. The transient analysis, on the other hand, is accurate but very expensive and difficult to implement because of its sensitivity to design change and schedule. The generalized shock spectra approach described in this paper, although still an approximate procedure, retains the advantages of low cost while maintaining a very reasonable accuracy. In addition, unlike Ref. 1, it is substantially less dependent upon the analysts' intuitive judgment in pairing spacecraft and launch vehicle modes. Because of its low cost, tuning effect of launch vehicle modes with spacecraft mode can be very easily explored and used for design to establish worst case. The procedure is currently being applied to the Galileo spacecraft loads analysis with very acceptable results.

Acknowledgments

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